

Black hole thermodynamics under the microscope

Kevin Falls and Daniel F. Litim

Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, U.K.

A coarse-grained version of the effective action is used to study the thermodynamics of black holes, interpolating from largest to smallest masses. The physical parameter of the black hole are linked to the running couplings by thermodynamics, and the corresponding equation of state includes quantum corrections for temperature, specific heat, and entropy. If quantum gravity becomes asymptotically safe, the state function predicts conformal scaling in the limit of small horizon area, and bounds on black hole mass and temperature. A metric-based derivation for the equation of state, and quantum corrections to the thermodynamical, statistical, and phenomenological definition of entropy are also given. Further implications and limitations of our study are discussed.

I. INTRODUCTION

The discovery of black hole thermodynamics nearly half a century ago points towards a deep connection between the laws of classical general relativity, thermodynamics and quantum mechanics due to the presence of causal horizons. Under the assumption that Einstein's equation holds true, it was noted that black hole solutions obey a set of laws analogous to those of thermodynamics [1] with the surface gravity κ and the area of the black hole horizon A playing the rôles of temperature T and entropy S , respectively [2]. Including quantum mechanics, the quantitative link between area, entropy, Newton's coupling G_N and Planck's constant \hbar was found to be

$$S = \frac{A}{4 G_N \hbar} \quad (1)$$

based on the thermal radiation emitted by quantum fields on black hole space-times [3]. Even more intriguingly, it was also realized that the thermodynamic structure of gravity constitutes an equivalence, meaning that the line of reasoning can be reverted: provided that the fundamental thermodynamical relation $\delta Q = T \delta S$ between heat, temperature and entropy holds true for every local Rindler causal horizon, Einstein's equation follows as the corresponding equation of state [4]. Hence, the semi-classical picture would seem to suggest that there exists an underlying micro-structure of space-time analogous to how the thermodynamics of a gas follows from the “coarse-graining” of atoms or molecules. This brings up intriguing questions including whether metric gravity should be quantized at all, or rather thought of as an emergent macroscopic phenomenon [5–8].

The Hawking effect also seems to introduce a new level of uncertainty into the laws of physics [9]. In particular a thermal bath of particles, as seen by distant observers, contains little information of the matter that initially collapsed to form the black hole and if left alone the black hole will eventually evaporate completely. This process suggests that pure states can evolve into mixed states, which is in conflict with the basic principle of a unitary time evolution in quantum mechanics. It would thus be important to understand the deeper origin of the Bekenstein-Hawking entropy (1) in terms of microscopic degrees of freedom, and how fluctuations will affect the thermodynamical structure of gravity.

The notion of “coarse-graining”, as put forward by K. Wilson [10, 11] and L. Kadanoff [12] nearly half a century ago, has become a key concept in the understanding of quantum field theory and statistical physics. On the level of the path integral the idea corresponds to the successive integrating-out of momentum modes. A virtue of these methods is that they permit a continuous interpolation between the microscopic and macroscopic degrees of freedom, controlled by exact

functional renormalisation group equations for the effective action [13, 14]. For gravity, the idea of a coarse-grained path integral has received increasing attention in the past decade [15–23], largely motivated by the quest for a gravitational fixed point within S. Weinberg’s asymptotic safety conjecture [24]. Applications to black holes cover RG-improved versions of Schwarzschild space-times in four dimensions [25] and their dynamics [26], the thermodynamics of higher-dimensional black holes [27] and their phenomenology [28], rotating black holes [29], black holes in higher-derivative gravity [30], and RG flows from boundary terms [31].

In this paper we follow a novel route to explore the idea that a continuous coarse-graining of metric degrees of freedom could give rise to the thermodynamics as encoded in the horizon structure of black hole solutions. The questions we wish to address with this is whether there exists a version of black hole thermodynamics applicable for small, possibly Planck-size, black holes, and if so, what can be learnt from the corresponding equation of state? Following closely the original derivation of black hole thermodynamics, our primary new addition is to replace the underlying action by a scale-dependent, coarse-grained effective action. Thermodynamics then provides a link between the scale-dependence of couplings and the horizon area of black holes, leading to quantum corrections for black hole state functions. This allows for a continuous interpolation between macroscopic black holes, where semi-classical results such as (1) serve as a reference point, and microscopic ones, which are informed by quantum corrections through the RG evolution of couplings. Our results are obtained in a metric-independent fashion and apply for non-rotating, rotating, or charged black holes alike. We also show that a metric, which carries the same equation of state, can be obtained by means of an RG improvement of semi-classical space-times, thereby closing a gap in the literature. In addition, we study the implications for black holes within asymptotically safe quantum gravity and derive quantum corrections to temperature and specific heat. Quantum corrections to the entropy are equally computed including those for a thermodynamical, statistical, and phenomenological definition of entropy, and contrasted with results from other approaches to quantum gravity. We also find that the conformal scaling of asymptotically safe gravity is encoded in the state function for small horizon areas.

The remaining part of the paper is organised as follows. We briefly recall the basics of black hole thermodynamics and the functional renormalisation group for gravity, and introduce notation and conventions (Sec. II). This is then followed by the construction of an RG improved version of black thermodynamics at the example of Kerr-Newman type black holes within Einstein-Maxwell gravity (Sec. III). We then specify to the case where gravity becomes weaker at high energies as predicted by the asymptotic safety conjecture, and discuss RG corrections to temperature, specific heat, entropy, and the occurrence of conformal scaling (Sec. IV). We also show that our set-up can be realized in terms of explicit RG improved black hole metrics, and discuss our results for the entropy and highlight similarities and differences with earlier studies (Sec. V). We end in Sec. VI with a brief discussion of results and some conclusions.

II. PRELIMINARIES

In this section we provide some prerequisites for our study, including a brief overview of black hole thermodynamics and Wilson’s renormalisation group. We also introduce some notation.

A. Black hole thermodynamics

In four dimensions stationary black hole solutions to the coupled Einstein-Maxwell equations are parameterised by their mass M , angular momentum J and charge q [32, 33], a result known as black hole uniqueness. This set of most general black hole solutions for long-ranged forces,

the Kerr-Newman black holes, are expected to be the end points of gravitational collapse [34]. It follows from black hole uniqueness that the area of the horizon may be considered as a function $A = A(M, J, q)$.

If an infinitesimal amount of matter crosses the horizon the area A of the horizon will vary according to [1]

$$\frac{\kappa}{8\pi G_N} \delta A = \delta M - \Omega \delta J - \Phi e \delta q \quad (2)$$

where κ , Ω and Φ are the surface gravity, angular velocity and electric potential evaluated at the horizon. Here G_N and e denote Newton's constant and elementary electric charge, respectively (we work in units $c = 1$). In this paper we make the split $q \rightarrow eq$ where q is the quantity of charge and e is the coupling. The equation (2) has the form of the first law of thermodynamics $\delta U = \delta Q + \mu_i \delta N_i$ for which the internal energy U is associated to M , the heat crossing the horizon δQ is identified with $\frac{\kappa}{8\pi G_N} \delta A$ and the conserved quantities N_i and the associated chemical potentials μ_i with $\{J, q\}$ and $\{\Omega, \Phi\}$, respectively. As in conventional thermodynamics one can think of the black hole area A as a "state function" $A(M, J, q)$, defining a set of states parameterised by M , J and q . Then by taking appropriate derivatives in line with the first law (2), one can obtain the intensive quantities κ , Ω and Φ . For an equilibrium thermodynamical process at temperature T , the heat transfer δQ due to the coarse-grained microscopic degrees of freedom is related to the change in entropy δS by

$$\frac{\delta Q}{T} = \delta S. \quad (3)$$

Additionally the second law of thermodynamics states that the entropy of an isolated system can never decrease $\delta S \geq 0$. By considering a Gedanken experiment in which some hot gas is thrown into a black hole, J. Bekenstein conjectured [2] that a black hole should itself have an entropy proportional to its horizon area $S \propto A$ in order that the second law of thermodynamics is not violated. Shortly after this, S. Hawking [3] showed, by studying a quantum field theory on a classical black hole space-time, that black holes will actually emit thermal radiation with a temperature $T = \hbar \frac{\kappa}{2\pi}$. Thus, identifying the heat flow of some microscopic degrees of freedom at a temperature $\hbar \frac{\kappa}{2\pi}$ crossing the horizon to be

$$\delta Q = \delta M - \Omega \delta J - \Phi e \delta q \quad (4)$$

the first law of black hole thermodynamics implies that the entropy of the black hole is given by (1). From now on we will use units $\hbar = 1$. S. Hawking's original derivation of the black hole entropy (1) centrally relied on a thermodynamical reasoning and a semi-classical approximation for quantum gravity. Subsequently it was shown by G. Gibbons and S. Hawking [35] that it is also possible to obtain these results directly from the Euclidean path integral for quantum gravity by taking the Einstein-Hilbert action (with vanishing cosmological constant and appropriate boundary terms) as the saddle point approximation. Thus, the entropy (1) also corresponds to the correct statistical entropy within this approximation to the full path integral.

B. Coarse-graining and the renormalisation group

The renormalisation group is a powerful tool to study the scale- or energy-dependence of quantum field theories and statistical systems. The essential idea in the construction of non-perturbative renormalization group equations, as put forward by L. Kadanoff and K. Wilson [10, 11], is to integrate-out the short-distance fluctuations, gradually, ordered according to their characteristic energy by means of a momentum cut-off k . As such, Wilson's approach leads to a coarse-grained

version of quantum field theory which continuously interpolates between the micro- and macro-physics [13]. In modern formulations, the coarse-graining is achieved by adding an infrared cutoff R_k at momentum scale k to the propagators which, within a few constraints, can be chosen freely [36–38]. The effect of this procedure is to generate a scale-dependent or “flowing” effective action Γ_k [39] which interpolates between a microscopic action at large RG scale ($k \rightarrow \infty$), and the full quantum effective action in the long-distance limit ($k \rightarrow 0$) [13]. Most importantly, the effective action obeys an exact functional identity [40]

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \quad (5)$$

which relates an infinitesimal change of Γ_k at RG scale $t = \ln k$ to a momentum trace over the propagator. For a simple mass term $R_k \propto k^2$ the RG flow (5) reduces to the well-known Callan-Symanzik equation. By construction, the inverse of the RG momentum scale is interpreted as the linear “resolution”

$$\ell \approx 1/k_{\text{phys}} \quad (6)$$

of the RG “microscope” at which the physics is observed [13, 39]. Ultimately this is a consequence of the RG flow (5) being local in field- and momentum-space, meaning that the change of the effective action at energy scale k is induced by the fluctuations of the quantum fields at about that energy scale.

For gravity, these ideas have been put forward in [15, 16, 41] (see [17–23] for reviews), primarily to provide a computational framework within which S. Weinberg’s asymptotic safety conjecture for gravity can be addressed [24]. The set of scale-dependent gravitational actions $\Gamma_k[g_{\mu\nu}]$ then describes the “quantum spacetime” obtained from integrating-out gravitational fluctuations down to the energy scale k . The coarse-grained metric field $\langle g_{\mu\nu} \rangle_k$, which solves the effective equations of motion

$$\frac{\delta \Gamma_k}{\delta g_{\mu\nu}} = 0, \quad (7)$$

has the interpretation of a Riemannian manifold averaged over the length scale (6) [42–44]. The RG trajectory $k \rightarrow \Gamma_k$ encodes how the appearance of the physical system, characterised by the effective metric field $\langle g_{\mu\nu} \rangle_k$, changes with the resolution of the RG “microscope” [42].

Many applications of the gravitational RG flow have dealt with the search for ultra-violet (UV) fixed points, a prerequisite for metric gravity to become a well-defined local quantum field theory at high energies following the asymptotic safety conjecture [17–19, 24]. By now, evidences for UV fixed points in gravity have been found in four-dimensional Einstein-Hilbert gravity [16, 41, 45–49], higher-dimensional gravity [41, 50, 51], higher-derivative gravity [52, 53], and coupling to matter fields [54–57]. A gravitational fixed point is also consistent with results from holography [58, 59], conformal reductions [60], Lorentzian signature [61], and minisuperspace approximations [62]. Phenomenological implications of a gravitational fixed point have been explored for black holes [25–31], cosmology [63–75], and particle physics [76–80]. Non-local low-energy corrections to the gravitational effective action have equally been addressed [81, 82].

In the remaining part of this paper, we adopt the RG and the scale-dependent gravitational action Γ_k to develop an RG improved version to the laws of black hole thermodynamics.

III. BLACK HOLES UNDER THE MICROSCOPE

In this section we introduce our set-up to implement quantum corrections to the thermodynamics of black holes using a continuous Wilsonian renormalisation group.

A. Action

We are interested in a four-dimensional theory involving gravity, $U(1)$ gauge fields, and possibly matter fields. In the spirit of a scale-dependent effective action we describe their dynamics in terms of the “flowing” Einstein-Hilbert action coupled to photons and matter, approximated by

$$\Gamma_k[g_{\mu\nu}, A_\mu] = \int d^4x \sqrt{-\det g_{\mu\nu}} \left[\frac{1}{8\pi G_k} \mathcal{R} + \frac{1}{4\alpha_k} F^{\mu\nu} F_{\mu\nu} \right] + S_m. \quad (8)$$

Here, \mathcal{R} denotes the Ricci scalar and F the field strength of the photon, and S_m stands for a possibly scale-dependent matter action. The effective action differs from the classical Einstein-Hilbert action coupled to matter in that all couplings are considered as running couplings whose tree level approximation describes the quantum effects of modes down to the energy scale k . It is understood as a solution to the RG flow for the Einstein-Maxwell theory in its domain of validity. In the deep infrared limit where the RG scale is removed ($k \rightarrow 0$) both the running Newton coupling G_k and the running fine structure constant $\alpha_k \equiv e_k^2/(4\pi)$ will approach their low-energy values $G \approx 6.674 \times 10^{-11} \text{ N (m/kg)}^2$ and $\alpha \approx \frac{1}{137}$. We assume that the scale-dependence of Newton’s coupling G_k and of the fine structure constant are known, at least approximatively, though the actual form of these functions is not important for our line of reasoning.

For large k , we will approach a fine grained action for high momentum modes. In perturbative quantum gravity, the action (8) would then cease to be a good approximation due to the non-renormalizability of gravity. On the other hand, if metric quantum gravity becomes asymptotically safe, the action persist towards higher energies [16, 41, 45–47]. The RG flow of euclidean Einstein-Hilbert gravity coupled to a $U(1)$ gauge field has recently been considered in [83].

B. Black holes and entropy

At fixed k , and by varying Γ_k with respect to the metric and the gauge fields we recover the Einstein-Maxwell theory coupled to an energy momentum tensor $T_m^{\mu\nu}$ and a current J^μ obtained from the matter action S_m . Setting $J^\mu = 0$ and $T_m^{\mu\nu} = 0$ Kerr–Newman-type black holes are the unique stationary black hole solutions. The sole difference with the standard solutions is that the couplings G_k and α_k explicitly take k -dependent values. As such we have a family of Kerr-Newman black hole solutions characterised by a fundamental relation between its mass M , the horizon area A , charge q , and angular momentum J , and the RG scale k . This relation has the form

$$A = A(M, J, q; k) \quad (9)$$

where the scale-dependence enters the equation only implicitly via the couplings G_k and e_k^2 . The equation (9) expresses an on-shell relation with respect to the underlying action Γ_k . The scale k indicates that degrees of freedom with momenta above k have been integrated out to give rise to a semi-classical space-time geometry. It is our assumption that these microscopic degrees of freedom also give rise to the thermodynamical properties of space-time. Under this assumption we think of their black hole entropy

$$S_k = \frac{A}{4G_k} \quad (10)$$

as accounting for those degrees of freedom which have already been integrated out from the path integral. It is worth noting the parametric dependence of (10) on G_k , which states that the entropy per area increases with decreasing gravitational coupling $G_k \rightarrow 0$, and vice-versa. With the area A

given by (9) the relation (10) will give an on-shell expression for the entropy $S_k = S_k(M, J, q; k)$. We could also consider an off-shell definition for the entropy where it is not assumed that the area is given by (9), but instead take (10) as the Wald entropy [84] obtained from the underlying action (8). Consequently the entropy would depend on the metric, via A and, independently, on the scale k . The RG flow for the off-shell entropy (10) taken at constant area is then given by

$$\frac{\partial}{\partial \ln k} S_k = -S_k \frac{\partial \ln G_k}{\partial \ln k} \quad (11)$$

and only depends on the RG flow of G_k , and not on the on-shell relation (9). We can think of this flow for the entropy as the “focusing of the microscope” through which the physics is viewed, in contrast to a change of the underlying state of the system which would additionally lead to a variation of the area δA . The family of Kerr-Newman black holes with (9) obeys the standard laws of black hole thermodynamics for all k . This is so because the thermodynamical nature of black hole solutions to (8) is independent of the actual numerical values of the couplings. These relations are modified as soon as the RG scale k is linked to the physical parameters of the black hole solution, to which we turn next.

C. Scale identification

In order to develop a renormalisation group improved version of black hole thermodynamics, we identify the degrees of freedom responsible for the thermodynamical properties of the black hole with those that have been integrated out in the underlying path integral. To this end we adjust the RG scale to the physical parameters of the black hole. For asymptotically large black holes the effective action approaches the classical infrared limit, and the relevant RG scale becomes very small $k \rightarrow 0$. For finite-size black holes, lesser modes are required in the underlying path integral to constitute the background geometry. The relevant RG scale at which to evaluate the effective action (8) should then be finite $k > 0$, and the value of the running couplings may be different from their infrared values. We will thus assume that there exists an “optimal” RG scale $k = k_{\text{opt}}$ at which to evaluate the couplings, set by the macroscopic spacetime geometry with black hole parameters M , J , and q ,

$$k = k_{\text{opt}}(M, J, q). \quad (12)$$

Heuristically, if the RG scale is taken much larger than k_{opt} , the effective action Γ_k is not yet a good tree level approximation for a black hole solution with physical parameters M , J and q , and additional quantum (loop) corrections will have to be taken into account. On the other hand, for k much smaller than k_{opt} the effective action and its saddle point solution may become too coarse-grained.¹ Under this assumption we will again have a set of Kerr-Newman-type black holes parameterised by M , J and q , except that now the space of black hole solutions is deformed by the underlying RG trajectory through the link (12). As a result, a new state function

$$A = A(M, J, q) \quad (13)$$

is obtained by inserting $k = k_{\text{opt}}(M, J, q)$ into (9) which, in general, may be different from the classical state function. Below we show that the scale $k_{\text{opt}}(M, J, q)$ is fixed up to an overall normalisation, provided that the black holes obey a scale-dependent version of black hole thermodynamics.

¹ This line of reasoning is similar to an optimized scale identification used in the context of inflation [65].

In order to achieve this goal we must decide on the appropriate generalisation for the variation of the entropy δS . We will take this variation as

$$\delta S_{k_{\text{opt}}} = \frac{\delta A}{4G_{k_{\text{opt}}}}. \quad (14)$$

Note that this corresponds to taking the partial derivative with respect to A . This choice amounts to a variation of the off-shell entropy with respect to the metric field at fixed RG scale k . This is similar to how the equations of motion are obtained from Γ_k , and ensures that we compare entropies which are defined with respect to the same coarse-graining scale. If, on the other hand, we are taking the full exterior derivative of $S_{k_{\text{opt}}}$ we would instead gain an extra term originating from the flow (11), giving

$$\delta S_{k_{\text{opt}}} = \frac{\delta A}{4G_{k_{\text{opt}}}} - S_{k_{\text{opt}}} \delta \ln G_{k_{\text{opt}}}. \quad (15)$$

The interpretation of this quantity is that it compares two different entropies defined relative to two distinct coarse-graining scales. In the spirit of our construction, we will therefore take (14) in favour of (15). We will come back to this in Sec. V C below.

D. Thermal equilibrium

Next we determine the scale (12) entering the relation (9) using a thermodynamical bootstrap. Assuming that (12) is given as a function of M , J and q we perform a Gedanken experiment and allow a small amount of matter to fall into the black hole which thereby will change in mass, charge, and angular momentum to settle down into a new state corresponding to the mass $M + \delta M$, angular momentum $J + \delta J$ and charge $q + \delta q$. This process induces a change in the scale (12) into $k_{\text{opt}} + \delta k_{\text{opt}}$. In order to describe this process thermodynamically we have to relate the change in heat with the change in entropy. We will assume that the relation

$$\frac{\delta Q}{T} = \delta S_{k_{\text{opt}}} \quad (16)$$

holds true, with the variation in entropy taken as (14). In the light of the results by T. Jacobson [4], the equation (16) has a natural interpretation as a RG improved form of Einstein's equations on the black hole horizon. In addition, and on general thermodynamical grounds we expect that a thermal description of the black hole embodied by the relation (16) should be valid provided the entropy and the specific heat are large [85],

$$\begin{aligned} 1/S &\ll 1 \\ \left| \frac{\partial T}{\partial M} \right|_{J,q} &\ll 1. \end{aligned} \quad (17)$$

We now turn to the heat δQ crossing the horizon which is given by

$$\delta Q = \delta M - \Omega \delta J - \Phi e_{k_{\text{opt}}} \delta q. \quad (18)$$

The heat is understood as the energy carried by the coarse-grained degrees of freedom with energy larger than (12). These are the degrees of freedom that have been integrated out in the path integral to obtain the effective equations of motion, in analogy to the “integrating-out” of individual atoms

or molecules which carry heat in a standard thermodynamical description of a gas. To continue, we note that the total change in the area of the black hole is given by

$$\delta A = 4G_{k_{\text{opt}}} \frac{2\pi}{\kappa} \delta Q + \left. \frac{\partial A(M, J, q; k)}{\partial \ln k} \right|_{k=k_{\text{opt}}} \frac{\delta k_{\text{opt}}}{k_{\text{opt}}}. \quad (19)$$

The first term follows from (2) since at constant k we obtain the classical variation of the area. The second term takes the implicit scale-dependence of A into account. It is proportional to the RG β -functions of the couplings and therefore accounts for the quantum corrections. These new terms imply that we go off-shell with respect to the equations of motion at scale k_{opt} to obtain a solution to the equations at a scale $k_{\text{opt}} + \delta k_{\text{opt}}$. In order to identify the scale k_{opt} which appears in (19) we rearrange this expression for δQ and insert it into the RHS of (16). With the LHS given by (14) we obtain the relation

$$\left(1 - \frac{2\pi}{\kappa} T\right) \delta A = \left. \frac{\partial A(M, J, q; k)}{\partial \ln k} \right|_{k=k_{\text{opt}}} \frac{\delta k_{\text{opt}}}{k_{\text{opt}}}. \quad (20)$$

The significance of (20) is as follows. The classical relation between temperature and surface gravity $T = \frac{\kappa}{2\pi}$ holds true provided the RHS vanishes. In the presence of RG corrections, the RHS describes corrections to the temperature of the black hole. Most importantly, we note that δk_{opt} must be proportional to δA independently of the specific form for the heat δQ . This implies that the scale k_{opt} depends on M, J and q only through the combination

$$k_{\text{opt}}(M, J, q) \equiv k_{\text{opt}}(A(M, J, q)). \quad (21)$$

Thus we are lead to the conclusion, via a thermodynamical argument, that the black hole area A is the unique scale associated to the black hole geometry which determines the renormalisation group scale $k_{\text{opt}}(A)$. Dimensional analysis then dictates that this relation reads

$$k_{\text{opt}}^2 = \frac{4\pi}{A} \xi^2 \quad (22)$$

where the factor 4π , the surface of the unit 2-sphere, is conventional and ξ is an undetermined dimensionless constant. The scale identification (22) has a straightforward generalization to dimensions different from four.

The thermodynamical bootstrap fixes the relation (22) only up to an unknown proportionality factor. This is reminiscent of the standard laws of black hole thermodynamics being independent of the numerical values of couplings. The occurrence of the factor ξ is understood from the RG point of view as the freedom of choosing the normalisation for k , which comes about via the Wilsonian momentum cutoff R_k . Hence the coefficient $\xi = \xi(R_k)$ depends on the RG scheme inasmuch as the value of $k_{\text{opt}} = k_{\text{opt}}(R_k)$ depends on it, to ensure that the effective physical cutoff scale $k_{\text{phys}} \approx k_{\text{opt}}/\xi$ is scheme-independent. For physical choices of the RG scheme we expect ξ to be of order unity, and assuming that this has been done we will set $\xi = 1$ for the remaining part of the paper.

In this light, the result (22) states that the underlying effective action Γ_k , (8), should be evaluated at the RG scale k_{opt} set by the horizon area of its black hole solution. In particular, since quantum fluctuations of momentum modes larger than k_{opt} have indeed been integrated out, the black hole area acts as a diffeomorphism invariant infrared cutoff for its effective action. This result is consistent with the view that thermodynamic properties originate from those degrees of freedom which constitute the black hole.

E. RG thermodynamics

We are now in a position to define the renormalisation group improved relation (13) between the area A and the physical parameters M , J and q by replacing the classical couplings by running couplings evaluated at the scale (22). This is most neatly expressed in terms of a mass function $M = M(A, J, q)$, with

$$M^2 \equiv \frac{4\pi}{A} \left[\left(\frac{A + 4\pi G_{\text{opt}}(A) e_{\text{opt}}^2(A) q^2}{8\pi G_{\text{opt}}(A)} \right)^2 + J^2 \right]. \quad (23)$$

It defines initial and final states of a thermodynamical process, in conjunction with a small RG transformation. The mass function is obtained from the standard relation for the Kerr-Newman black hole by replacing the classical couplings with $G_N \rightarrow G_{\text{opt}}(A) \equiv G_{k_{\text{opt}}(A)}$ and $e^2 \rightarrow e_{\text{opt}}^2(A) \equiv e_{k_{\text{opt}}(A)}^2$ under the identification (22). The relation (23) then allows us to parameterise these states simply by the mass M , charge q , and angular momentum J thus recovering a RG improved version of black hole uniqueness. Solving for A we find RG improved state functions $A(M, J, q)$. If there are several roots A_i for the same values of M , J and q these have the natural interpretation as multiple horizons for the same black hole e.g. inner and outer horizons of a Kerr black hole as in the classical theory. Note that since these horizons generically have different entropies and temperatures, being in thermal equilibrium with either of them corresponds to a different thermodynamical state. Their entropy is given by

$$S_{k_{\text{opt}}} = \frac{A}{4G_{\text{opt}}(A)} \quad (24)$$

with its thermodynamical variation given by (14). At this point it is useful to remember that the scale k tells us which degrees of freedom have been integrated out in the path integral and that the relation (22) is obtained by requiring that k is optimised according to the background geometry. So the entropy (24) counts the number of degrees of freedom that have been integrated out in this optimal coarse graining.

The temperature T , angular velocity Ω and electric potential Φ appear in an improved first law of black hole thermodynamics obtained by putting the variation of the entropy (14) on the RHS of (16) and (18) on the LHS, leading to

$$T \frac{\delta A}{4G_{\text{opt}}(A)} = \delta M - \Omega \delta J - \Phi e_{\text{opt}}(A) \delta q. \quad (25)$$

This differs from the standard first law by the presence of the area-dependent couplings. Also, the relation between temperature and the classical expression for the surface gravity of the black hole receives RG corrections. (We will see in Sec. V that there exist explicit RG-improved metrics for which (25) holds true with the temperature identified with the surface gravity felt by a test particle on these black hole metrics.) The intrinsic quantities T , Ω and Φ are obtained by taking derivatives of M (or A) in line with (25). The RG improved black hole temperature is obtained as

$$\frac{1}{T} = \frac{1}{4G_{\text{opt}}(A)} \frac{\partial A}{\partial M} \quad (26)$$

which receives corrections containing derivatives of the couplings and their RG β -functions. On the other hand both Ω and Φ can be simply obtained from their classical expressions by replacing the classical couplings by the functions $e_{\text{opt}}(A)$ and $G_{\text{opt}}(A)$. This ‘factorization’ holds true since

derivatives of (23) with respect to J and q , by the virtue of (22), cannot touch the running couplings as they only depend on the area A .

We note in passing that if we had instead used (15) in the RHS of (16) we would obtain a different temperature

$$\frac{1}{T} = \frac{1}{4G_{\text{opt}}(A)} \frac{\partial A}{\partial M} \left(1 - \frac{\partial \ln G_{\text{opt}}(A)}{\partial \ln A} \right). \quad (27)$$

In particular this would imply that if $G(A) \propto A$, the temperature would diverge due to the vanishing of the bracket on the RHS. In this paper we always take (14) to define the variation of the entropy leading to (26), where no such divergence appears.

At a practical level the formalism presented here allows us to obtain models of quantum black hole thermodynamics given an RG trajectory for G_k and e_k . This provides a controlled way to include quantum corrections without moving too far from the semi-classical thermodynamics of black holes. Ultimately such a (thermal) description may break down at high energies where we expect that the action (8) should include higher order terms and where the thermodynamical approximation based on Kerr-Newman black holes will no longer be a good one.

F. Semi-classical limit

For low energies we must recover classical general relativity such that astrophysical black holes are described by the Einstein-Maxwell equations. This is achieved provided we have an RG trajectory with the limits

$$\begin{aligned} G_k &\rightarrow G_N & \text{for } k &\ll M_P \\ e_k^2 &\rightarrow e^2 & \text{for } k &\ll m_e \end{aligned} \quad (28)$$

where $M_P = 1/\sqrt{G_N}$ is the Planck mass and m_e is the electron mass. For Newton's coupling, the limit of classical general relativity is achieved as a consequence of IR attractive fixed points. The scale identification (22) implies that these limits are achieved for a black hole with a sufficiently large area A as this entails that the underlying effective action is integrated down to $k \rightarrow 0$. Consequently, astrophysical black holes will then be described accurately by classical general relativity. Our model of RG improved black hole thermodynamics then passes the first mandatory test of recovering the right semi-classical limit in the infra-red. We note that since $M_P \gg m_e$ there exists a large range of scales for which gravity remains semi-classical, but where the running of e_k will induce quantum corrections to tiny charged black hole as soon as the radius of the black hole approaches the Compton wavelength of an electron.

IV. THERMODYNAMICS AND ASYMPTOTIC SAFETY

Our reasoning in the previous sections was independent of the actual form of the running couplings G_k and α_k and, therefore, of the UV completion of gravity. In this section we consider an explicit example where gravity becomes anti-screening at short distances as predicted by the asymptotic safety conjecture for gravity [24].

A. Fixed point and characteristic energy

Asymptotic safety is a non-perturbative generalisation of asymptotic freedom for which the relevant couplings of a theory reach a non-Gaussian fixed point at high energies. If realised in

Nature, the asymptotic safety conjecture implies that the short-distance fluctuations of gravity shield the theory from the divergences of standard perturbation theory [24]. Furthermore, to ensure predictivity, the number of relevant directions flowing away from the fixed point must be finite. There has recently been much evidence that this is the case of gravity [16, 41, 45, 46, 52, 53]. For Newton's constant this implies that the dimensionless coupling $g_k = k^2 G_k$ reaches a fixed point $g_k \rightarrow g_* \neq 0$ in the UV limit, implying near-conformal behaviour with a characteristic weakening $G_k \rightarrow g_*/k^2$ at short distances. The presence of such an UV fixed point also implies that gravity may exist as a QFT to arbitrarily large energy scales and is therefore non-perturbatively renormalisable.

To explore the implications of the asymptotic safety conjecture for the physics of black holes we allow for a non-trivial scale-dependence of Newton's constant. In terms of the graviton anomalous dimension $\eta = \frac{d \ln G_k}{d \ln k}$, the RG flow reads [17–19]

$$\frac{d G_k}{d \ln k} = \eta G_k. \quad (29)$$

In general, the anomalous dimension is a function of all couplings of the theory. In perturbation theory, one finds $\eta = -2\omega k^2 G_k + \mathcal{O}(G_k^2)$ where the sign of the one-loop coefficient ω depends on the field content of the theory. Gravity is perturbatively anti-screening if ω is positive. In the IR limit, the anomalous dimension and (29) are arbitrarily small meaning that $G_k \approx G_N$. At a non-trivial fixed point the anomalous dimension becomes large, $\eta = -2$, to ensure that the dimensionless gravitational coupling $G_k k^2$ approaches a non-trivial UV fixed point of gravity g_* . Analytical RG flows which interpolate between these limit have been given in [41]. For our purposes, a good approximation for the integrated RG flow is given by

$$\frac{1}{G_k} = \frac{1}{G_N} + \frac{k^2}{g_*}. \quad (30)$$

In the infrared limit the running coupling reduces to its classical value. In the UV limit the second term takes over leading to the asymptotic weakening of gravity $G_k \rightarrow 0$. Note that g_* plays a double rôle in the RG flow (30). In the IR limit, $1/g_*$ represents the perturbative one-loop coefficient ω . In the UV limit, g_* stands for the non-perturbative fixed point. In the full theory, these numbers can be different from each other. Typically one finds values g_* of order unity. For the purpose of this study, we shall keep g_* as a free parameter. Also, the flow (30) is non-perturbatively anti-screening as long as g_* is positive, in agreement with RG results for purely gravitational flows. In set-ups where the RG running of Newton's coupling is dominated externally, eg. by matter fields, the one-loop coefficient may turn negative. Returning to (30), we note that the quantum corrections are responsible for the appearance of a characteristic energy scale

$$E_c^2 = g_* M_P^2 \quad (31)$$

where we have introduced the Planck mass M_P , with $M_P^2 \equiv 1/G_N$. At the energy scale $k = E_c$ we have that the tree level term equals the quantum corrections in magnitude, and hence the scale E_c sets the boundary between IR and UV scaling. We also note that the quantum corrections are suppressed in the limit where $1/g_* \rightarrow 0$. The meaning of this limit is that the theory still owns an UV fixed point except that it is infinitely far away and cannot be approached within finite RG 'time' $t = \ln k$. This is equivalent to a semi-classical approximation with no RG running at all, corresponding to the limit $\hbar \rightarrow 0$.

B. Critical mass and area

We now show that an asymptotically safe RG running such as (30) with the cross-over scale (31), in conjunction with the reasoning of the previous section, lead to the appearance of a new

mass scale

$$M_c^2 = \frac{1}{g_*} M_P^2. \quad (32)$$

The scale M_c owes its existence to the presence of the fixed point g_* and is hence absent in the classical theory. It is qualitatively different from the classical Planck scale M_P which is an infrared parameter of Einstein gravity. Note also that M_c is dual to the energy scale E_c ,

$$M_c E_c = M_P^2 \quad (33)$$

irrespective of the value for g_* . Extended fixed point searches in pure four-dimensional quantum gravity indicate that g_* is of the order one, with $M_c \approx E_c$ and $M_c \approx M_P$. The classical limit is recovered by taking $1/g_* \rightarrow 0$ where the mass scale $M_c \rightarrow 0$ disappears.

The significance of the mass scale (32) can be understood from the following observations. (For simplicity we restrict the discussion to the case where $q = 0$.) We insert the running coupling (30) into (23) to find

$$M^2 = \frac{4\pi}{A} \left(\frac{(A + 4\pi G_N^2 M_c^2)^2}{64\pi^2 G_N^2} + J^2 \right). \quad (34)$$

This function encodes all the relevant information needed to obtain properties of the RG improved black hole via the first law (25). Note that it takes a form similar to the classical Kerr-Newman black hole (i.e. (23) with constant G and e) with M_c^2/M_P^2 playing the role of the classical charge $(eq)^2$. Taking the limit $M_c \rightarrow 0$ we obtain the classical Kerr black hole relation between the mass, area and angular momentum. Leaving M_c non-zero we can solve (34) to find the quantum-corrected area $A_{\pm}(M, J)$ of the outer and inner horizons of the RG improved black hole,

$$A_{\pm} = 4\pi G_N \left(2G_N M^2 - G_N M_c^2 \pm 2\sqrt{G_N^2 M^4 - J^2 - G_N^2 M_c^2 M^2} \right). \quad (35)$$

Taking a derivative of this expression with respect to the mass M one can find the temperature of the black hole T from the first law (25). Similarly one may find the angular momentum by taking a derivative with respect to J . When the expression inside the square root of (35) vanishes we have degeneracy between inner and outer horizons $A_+ = A_-$ and the temperature of the black hole falls to zero. This corresponds to an extremal black holes with mass

$$M_{\text{ex}}(J)^2 = \frac{1}{2} \left(M_c^2 + \sqrt{4J^2 + \left(\frac{M_c}{M_P} \right)^4} M_P^2 \right). \quad (36)$$

In the classical limit the extremal black hole mass (36) reduces to the extremal Kerr mass

$$M_{\text{Kerr}}^2(J) = |J| M_P^2. \quad (37)$$

The physical meaning of the mass scale (32) then follows from (36) in that it characterizes the mass of the smallest achievable black hole $M_c = M_{\text{ex}}(J = 0)$ with a causal horizon. Here, the existence of a lightest black hole is a direct consequence of the RG equations for G_k . As we probe gravity at smaller distances the anti-screening effects weaken the gravitational interactions such that a black hole horizon can no longer form, and the notion of a semi-classical black hole space-time ceases to exist. The horizon area of the smallest black hole is given by $A_c = 4\pi(G_N M_c)^2$, which can be written as

$$A_c = \frac{4\pi}{g_* M_P^2}. \quad (38)$$

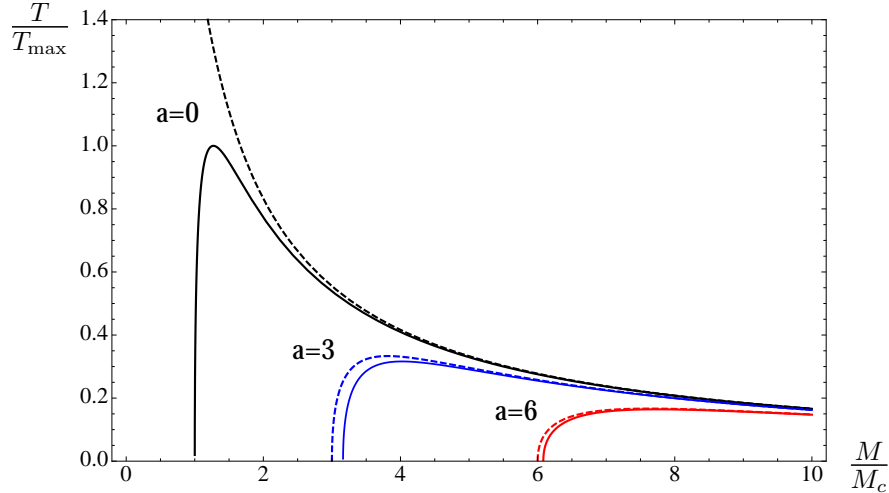


Figure 1: Horizon temperature as a function of the black hole mass, comparing classical gravity (dashed lines) with asymptotically safe gravity with $g_* = 1$ (solid lines) for several angular momenta, with a given in units of $1/M_c$. Temperatures are normalised to the maximum temperature of the asymptotically safe Schwarzschild black hole (see text).

Using (38) together with (22) identifies the RG scale corresponding to the smallest black hole as the cross-over scale (31), $k_{\text{opt}} = E_c$. We also note that for masses $M > M_c$ and vanishing angular momentum $J = 0$ an inner horizon of area A_- will always be present. This holds true independently of the detailed form of the RG equation (30), showing that the degeneracy of the Schwarzschild black hole, which classically does not display a Cauchy horizon, is lifted by asymptotically safe quantum gravity fluctuations.

C. Temperature and specific heat

The temperature T of the black hole follows from (34) or (35) through appropriate differentiation. In Fig. 1 we show the temperature (26) of the black hole for the outer horizon for various values of the rotation parameter $a = J/M$. In all cases, and in contradistinction to the classical Schwarzschild black hole, the temperature falls to zero in the limit $M \rightarrow M_c$. This pattern implies the existence of a maximum temperature which at $J = 0$ is found to scale as

$$T_{\text{max}} \propto \sqrt{g_*} M_P = E_c. \quad (39)$$

With (30) the proportionality factor reads $(1 + \sqrt{5})^{1/2}/(2^{3/2}(2 + \sqrt{5})\pi) \approx 0.024$ showing that the largest achievable temperature stays well below Planckian energies for all M , provided that g_* is of order unity. The specific heat associated to the black hole is defined as

$$C = \frac{\partial M}{\partial T}. \quad (40)$$

In Fig. 2 we show the specific heat (40) in comparison with the classical result (dashed lines) for different angular momenta. For vanishing angular momenta, the classical specific heat is always negative. Once RG effects are taken into account, the specific heat changes sign for black hole masses approaching M_c . This happens in a regime where the thermodynamical approximation is viable, and thus is a prediction of our theory. It implies a qualitative change in the thermodynamics in that the black hole becomes thermodynamically stable. The specific heat vanishes once its mass

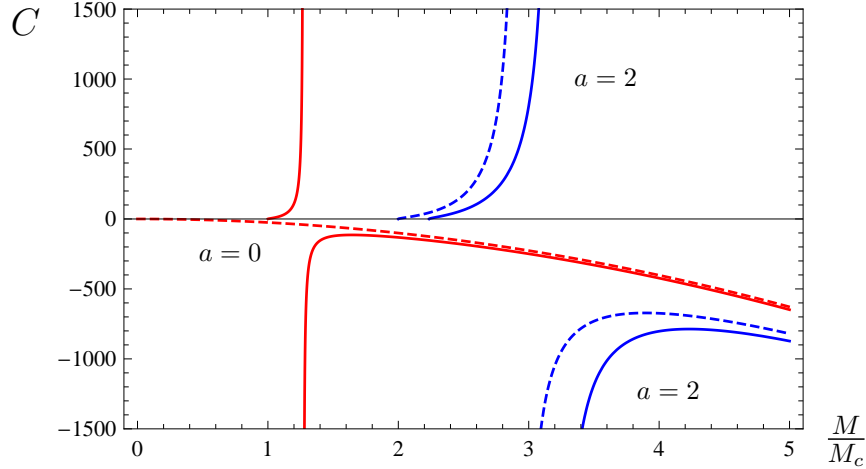


Figure 2: Specific heat as a function of the black hole mass, comparing classical gravity (dashed lines) with asymptotically safe gravity ($g_* = 1$, solid lines) for several angular momenta a , given in units of $1/M_c$ (see text).

is as low as $M = M_c$ allowing for a cold black hole remnant. Furthermore, for non-vanishing angular momenta, classical black holes show a change in specific heat for sufficiently small black hole masses. Including quantum corrections, we note that the sign flip in the specific heat takes place already at larger masses. Furthermore, the critical black hole mass is also larger than in the classical case.

D. Inverse mass expansion

It is interesting to perform an expansion in powers of M_c^2/M^2 , which corresponds to an expansion in powers of \hbar , see Sec. IV A. This is achieved by either sending $M_c \rightarrow 0$ at fixed black hole mass M , or by sending the mass to infinity $1/M \rightarrow 0$ at fixed M_c . We adopt the RG running (30). Expanding the state function in units of the classical state function, we find

$$\frac{A}{A_{\text{cl}}} = 1 - \frac{1}{2} \left(\frac{M_c}{M} \right)^2 - \frac{1}{16} \left(\frac{M_c}{M} \right)^4 - \frac{1}{4} \left(\frac{M_{\text{Kerr}}}{M} \right)^4 \left(\frac{M_c}{M} \right)^2 + \text{subleading}, \quad (41)$$

using (32) and (37). Here we have also introduced the area of the classical horizon A_{cl} , which for $J = 0$ reads $A_{\text{cl}} = 4\pi(2G_N M)^2$. Note that the expansion is a power series in M_c^2/M^2 times kinematical corrections in powers of M_{Kerr}^2/M^2 . All subleading terms originate from fluctuations and decrease the horizon area relative to the classical horizon at the same mass and angular momentum. The first two terms are independent of angular momentum. The ratio (41) interpolates between 1 in the classical limit and $\frac{1}{4}$ in the limit where the black hole becomes critical $M \rightarrow M_c$. Similarly, for the temperature we find

$$\frac{T}{T_{\text{cl}}} = 1 - \frac{1}{4} \left(\frac{M_c}{M} \right)^2 - \frac{5}{16} \left(\frac{M_c}{M} \right)^4 - \frac{5}{16} \left(\frac{M_{\text{Kerr}}}{M} \right)^4 \left(\frac{M_c}{M} \right)^2 + \text{subleading}, \quad (42)$$

showing that quantum corrections decrease the temperature in comparison to the classical one. Here, T_{cl} denotes the classical temperature of the black hole which reads $T_{\text{cl}} = M_P^2/(8\pi M)$ for $J = 0$. The corrections to (41) and (42) are algebraic, which is a consequence of the power-law running of Newton's coupling under the RG flow (30).

E. Conformal scaling

We now turn to the conformal scaling laws of black holes within asymptotically safe quantum gravity in the vicinity of an ultraviolet fixed point. Under the assumption that the underlying partition function at high energies is dominated by semi-classical black holes, it has been suggested by O. Aharony and T. Banks [86] and by A. Shomer [87] that a quantum theory of metric gravity may not exist as a local quantum field theory. Here, we re-evaluate this line of argument in the light of the asymptotic safety conjecture.

For want of generality we consider the case for black holes in general dimension d , and take $J = 0$ for simplicity. We recall that for a conformal field theory (CFT), the entropy and energy scale as

$$S \sim (RT)^{d-1}, \quad E \sim R^{d-1}T^d \quad (43)$$

where R is the radius of spacetime under consideration, and T is the temperature. It is important when dealing with black holes to note that the black hole radius R depends on the energy $E = M$ of the black hole. Therefore we should consider a relation between the entropy and energy densities of the form

$$\frac{S}{R^{d-1}} \sim \left(\frac{E}{R^{d-1}} \right)^\nu. \quad (44)$$

For a conformal field theory, the scaling behaviour (43) dictates (44) with

$$\nu_{\text{CFT}} = \frac{d-1}{d} \quad (45)$$

and $T^{d-1} \sim S/R^{d-1}$. The scaling relation (45) is different from the one put forward by A. Shomer [87], according to which entropy scales with energy as $S \sim E^{\frac{d-1}{d}}$. The latter would only be true if the radius was independent of the mass and entropy. This is not the case for black holes such as those considered here. For a semi-classical black hole we have that $A \sim R^{d-2}$, $E \sim G_N^{-1}R^{d-3}$ and $S \sim R^{d-2}G_N^{-1}$, where R is the Schwarzschild radius, leading to the scaling relation (44) with index

$$\nu_{\text{BH}} = \frac{1}{2} \quad (46)$$

for any dimension. Not surprisingly, (46) shows that (semi-)classical black holes do not behave as conformal field theories. This also follows from the fact that the Schwarzschild solution depends on the dimensionful quantity G_N , implying that the physics cannot be scale invariant. On the other hand, extrapolating down to two dimensions where G_N is dimensionless, we find that the semi-classical estimate (46) is formally in agreement with conformal scaling (45).

We now turn to the scaling of asymptotically safe black holes. The central observation is that the horizon area always scales according to $A \sim k^{2-d}$, and hence $R \sim k^{-1}$. For $k \ll E_c$, energy, entropy and temperature scale exactly the same way as in the classical case, leading to (46). For $k \gg E_c$, fixed point scaling takes over and we find that entropy becomes a constant while both mass and temperature scale linearly with energy $M \sim k$ and $T \sim k$ in this limit, leading to

$$T \sim R^{-1}, \quad E = M \sim R^{-1} \quad \text{and} \quad S = \text{const.} \quad (47)$$

for asymptotically safe black holes in any dimension. The scaling (47) is evidently conformal, obeying (44) with ν given by (45). The appearance of conformal scaling can also be understood by noting that the gravitational ultraviolet fixed point removes the infrared scale G_N from the

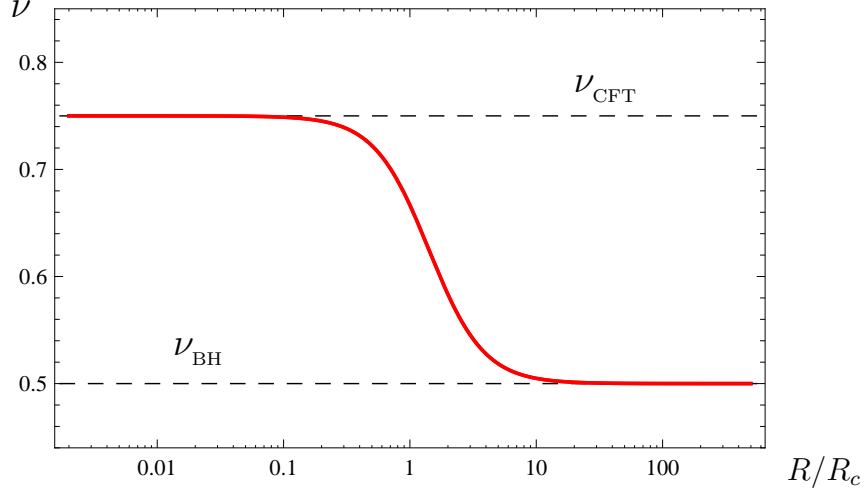


Figure 3: Scaling index for an asymptotically safe Schwarzschild black hole in four dimensions interpolating between the classical value ν_{BH} for large horizon radii and the conformal limit ν_{CFT} for small radii.

set-up. Consequently, in the absence of any other scales, the system must fall back onto (44) for any dimension. In Fig. 3, we have computed the index

$$\nu = \left(d - 1 - \frac{\partial \ln S}{\partial \ln R} \right) \left(d - 1 - \frac{\partial \ln E}{\partial \ln R} \right)^{-1}. \quad (48)$$

in four dimensions along the RG trajectory (30), with R_c denoting the scale corresponding to $A = A_c$ and $k = E_c$. With decreasing R , the index shows a smooth cross-over from classical behaviour for large R to conformal scaling for small R . Interestingly, this result is also consistent with a holographic view on the RG, see [58, 59]. We conclude that the UV fixed point scaling of asymptotically safe quantum gravity is encoded in the Cauchy horizon of its black hole solutions.

V. ENTROPY AND BLACK HOLE SPACE-TIMES

In this section we provide explicit space-time metrics which carry the thermodynamics derived in the previous sections.. We also relate our findings with earlier work based on RG improved metrics, discuss the entropy and its quantum corrections, and compare results with other approaches to quantum gravity.

A. Metrics

The construction of the previous sections makes no reference to an explicit underlying space-time metric. For some applications, it will be useful to have explicit RG improved metrics available which carry the thermodynamics derived above. In fact, it is possible to provide such metrics for any choice of coordinates. As an example, we consider the Kerr metric for an uncharged black hole ($q = 0$) in the familiar Boyer-Lindquist coordinates,

$$ds^2 = - \left(1 - \frac{2G M r}{\rho^2(r)} \right) dt^2 - \frac{G M r}{\rho^2} a \sin^2 \theta dt d\phi + \frac{\rho^2(r)}{\Delta(r)} dr^2 + \rho^2(r) d\theta^2 + \frac{\sin^2 \theta}{\rho^2(r)} [(r^2 + a^2)^2 - a^2 \Delta(r) \sin^2 \theta] d\phi^2 \quad (49)$$

where $a = \frac{J}{M}$ denotes the angular momentum in units of the mass, and

$$\Delta(r) = r^2 - 2GMr + a^2 \quad (50)$$

$$\rho^2(r) = r^2 + a^2 \cos^2 \theta. \quad (51)$$

The horizons radii are found from solving $\Delta(r_{\pm}) = 0$ with r_+ and r_- the well-known outer and inner horizon, respectively, and the horizon area is then given by $A = 4\pi^2(r_{\pm}^2 + a^2)$. In the classical theory G is a constant, given by Newton's coupling. In the spirit of a renormalization group improvement, we now wish to take the RG running of couplings into account, replacing

$$G \rightarrow G(r, \dots) \quad (52)$$

where the new coupling $G(r, \dots)$ depends on the coordinates and parameters of the space-time metric such as the radial distance r . We expect, by continuity, that changes in the numerical value of G in (49) along some RG trajectory account for the leading corrections to the effective space-time geometry. The RG equations provide us with the scale-dependence of couplings $G \rightarrow G(k)$, and the coordinate-dependence of couplings is then imported by means of a scale identification

$$k = k(r, \theta; a, M). \quad (53)$$

Thus, the scale identification (53) is a central ingredient because it affects the physical properties of RG improved black hole metrics. In a multiscale problem, deriving an suitable definition for k in terms of the physical mass parameters is a highly non-trivial task [25, 27, 88]. We claim, however, that the scale identification

$$k^2 \sim \frac{1}{r^2 + a^2} \quad (54)$$

is distinguished as it leads to an RG improved black hole space-time with identical thermodynamical relations as those derived in Sec. III in a metric-independent manner. The identification implies that one recovers (22) and hence $G(r_{\pm}) = G_{\text{opt}}(A)$ on the horizons $r \rightarrow r_{\pm}$.

B. Thermodynamics

We establish the thermodynamical equivalence between RG improved black hole metrics with (52) and (54) and the RG thermodynamics derived in Sec. III. The equivalence is such that the relation between M , J , q and A given by (23) is satisfied, and that the temperature (26) corresponds exactly to the surface gravity of the RG improved black hole metric, i.e. $T = \frac{\kappa}{2\pi}$. Our reasoning is independent of the specific RG scale dependence of couplings. We consider the example of the Kerr-Newman black hole, and begin by replacing the couplings through running couplings using (54). We denote them as $G(r)$ and $e^2(r)$, although they also depend on a . The RG improved equations for the Kerr-Newman black hole follow from the Kerr metric (49), substituting $2GMr$ by $2G(r)Mr - G(r)e^2(r)q^2$. The horizon condition at radial coordinate $r = r_+$ is now given by $\Delta(r_+) = 0$ where

$$\Delta(r) = r^2 + a^2 - 2G(r)Mr + G(r)e^2(r)q^2. \quad (55)$$

The area of the black hole event horizon reads $A = 4\pi(r_+^2 + a^2)$ in terms of the rotation parameter a and r_+ . From $\Delta = 0$ we have the relation

$$r_+ = \frac{A + 4\pi e^2(r_+)q^2 G(r_+)}{8\pi M G(r_+)}. \quad (56)$$

One then finds a state function which relates mass with angular momentum, charge and the area

$$M^2 = \frac{4\pi}{A} \left[\left(\frac{A + 4\pi e^2(r_+) q^2 G(r_+)}{8\pi G(r_+)} \right)^2 + J^2 \right]. \quad (57)$$

Upon the use of (54), and hence $G(r_+) = G_{\text{opt}}(A)$ and $e^2(r_+) = e_{\text{opt}}^2(A)$, we find that the state function (57) agrees with (23). Since the functional dependence of $M(A, J, q)$, as given by (23), on J and q is the same as for a classical black hole we find that the potentials Ω and Φ obtained by taking derivatives of M equally retain their classical form, the only difference being that e^2 and G_N are replaced by the running couplings, and the classical horizon radius replaced by r_+ . Expressed in terms of r_+ and a , the potentials

$$\Omega = \frac{\partial M}{\partial J} = \frac{a}{r_+^2 + a^2} \quad (58)$$

$$\Phi = \frac{1}{e(r_+)} \frac{\partial M}{\partial q} = e(r_+) q \frac{r_+}{r_+^2 + a^2} \quad (59)$$

agree with the expressions obtained from the metric and the RG improved electric potential. Finally, we turn to the black hole temperature. In the metric formulation it is given by the surface gravity on the black hole horizon $T = \frac{\kappa}{2\pi} \equiv \frac{1}{4\pi} \frac{\Delta'(r_+)}{r_+^2 + a^2}$. Using (55), we find that

$$T = \frac{1}{4\pi r_+} \left[\frac{r_+^2 - a^2}{r_+^2 + a^2} - \frac{r_+}{G(r_+)} G'(r_+) \frac{e^2(r_+) q^2 G(r_+)}{r_+^2 + a^2} \left(1 - \frac{r_+}{e^2(r_+)} e^{2'}(r_+) \right) \right], \quad (60)$$

where primes denote derivatives with respect to the argument. We have to show that this expression is equivalent to the temperature defined in (26), $T = 4G(A)\partial M/\partial A$. Using the mass function (23) as well as (56), we find explicitly

$$T = \frac{1}{4\pi r_+} \left[\frac{r_+^2 - a^2}{r_+^2 + a^2} - \frac{2r_+^2}{r_+^2 + a^2} \frac{\partial \ln G_{\text{opt}}}{\partial \ln A} - \frac{G_{\text{opt}} e_{\text{opt}}^2 q^2}{r_+^2 + a^2} \left(1 - \frac{2r_+^2}{r_+^2 + a^2} \frac{\partial \ln e_{\text{opt}}^2}{\partial \ln A} \right) \right] \quad (61)$$

Clearly, (60) and (61) agree in the absence of RG corrections. In the presence of non-trivially running couplings, the terms involving derivatives of couplings have to agree as well. Here, in consequence of the scale identification (22) and (54), we have that

$$r \partial_r|_{r=r_+} = \frac{2r_+^2}{r_+^2 + a^2} A \partial_A \quad (62)$$

when applied on the running couplings. Using (62) we therefore conclude that (60) and (61) are identical, term by term, as claimed.

It is worth pointing out that the thermodynamical consistency of (49) with (52), (54) is non-trivial. In fact, one cannot expect that (49) with (52) and a generic matching necessarily leads to a thermodynamically consistent picture. In the literature, physically motivated matching conditions have been explored including $k \sim 1/r$ [25, 27–30], or $k \sim r_{\text{cl}}^{\gamma-1}/r^\gamma$ for some model parameter γ [27], and matchings $k \sim 1/D$ [25–27] where $D(r, \theta)$ denotes the proper distance of the classical space-time. The RG improved metrics for all matchings studied thus far consistently predict the existence of a smallest Planck-size black hole. However, for rotating black holes, none of these obey (62) and the related metrics fail to reproduce (16) or equality of the temperatures (60) and (26). Moreover, in these cases one cannot define an entropy function without giving up the relation $T = \frac{\kappa}{2\pi}$ since the 1-form $\delta Q/T$ is neither exact nor an integrating factor can be found [29]. In turn, the scale identification (54) resolves these matters. For Schwarzschild black holes this aspect is hidden as the relation (62) becomes less restrictive. Then matchings of the form $k \sim 1/r$ lead to a consistent thermodynamics, and the 1-form $\delta Q/T$ is trivially exact.

C. Entropy

Next we turn to the entropy and its quantum corrections in the light of the RG, and compare the thermodynamical and statistical entropy with Clausius' definition, and with results from the literature based on other approaches.

In the absence of quantum gravity effects, the classical Bekenstein-Hawking result (1) states that the black hole's entropy is larger, at fixed area, the smaller the classical coupling G_N (and vice versa). Under the renormalisation group flow, the entropy is modified, (10). Within the asymptotic safety scenario, the key quantum gravitational effect is that the running Newton coupling decreases (increases) with increasing (decreasing) RG momentum scale, provided the fixed point g_* is positive (negative). Therefore we expect that the quantum corrections to the entropy have the same sign as g_* .

More specifically, inserting the non-perturbative RG running (30) into (24), and also using (22), the thermodynamical entropy reads

$$S = \frac{A}{4G_N} + \frac{\pi}{g_*}. \quad (63)$$

The result applies equally for non-rotating, rotating or charged black holes. The constant term is a fingerprint of the underlying fixed point. There are no subleading terms in inverse powers of the area but they could arise from more sophisticated approximations for the RG flow [41]. Unlike the Bekenstein-Hawking entropy (1), the quantum-corrected expression (63) remains strictly positive even for vanishing area due to the non-perturbative RG running of Newton's coupling and $g_* > 0$. The constant term has a quantum origin and contains an additional power of \hbar compared to the leading term. The entropy is dominated by the first term for large masses and horizon areas, and the quantum corrections are parametrically suppressed both as $\propto G_N/A$ and as $\propto \hbar$. In the limit where the mass approaches the critical mass M_c , the entropy approaches the value

$$S_c = 2\pi/g_*. \quad (64)$$

For g_* of the order one, as found in explicit RG studies, the entropy S_c of a critical black hole is of the order of a few. Hence, the numerical value of the gravitational fixed point determines the effective number of degrees of freedom of a quantum black hole with mass M_c . In the asymptotic limit of vanishing area, the theory becomes conformal and the entropy a constant of the order of a few, see Sec. IV E. This limit can also be achieved for parametrically small $g_* \rightarrow 0$ at fixed area, corresponding to a regime of quantum dominance where $1/\hbar \rightarrow 0$. We also note the absence of logarithmic corrections to the expression given in (63). This is so because the entropy expression (63) ultimately arises from summing over all horizon areas [89]. The additional area-dependence entering through the RG running of the Newton's coupling, responsible for the constant term, is not generating a logarithm, although it could have done so, provided that the gravitational RG running (29) explicitly receives logarithmic dependences on the RG scale.

If, on the other hand, we use Clausius' phenomenological definition for the entropy, it follows from (26) (which holds true for the RG improved metric) that

$$S = \int dS = \int \frac{dM}{T} = \int \frac{dA}{4G(A)}. \quad (65)$$

The total derivative of (65) leads to (14), and the entropy arises as a weighted sum over all areas, where the running of the inverse Newton coupling with area serves as the weighting factor. Clausius' rule assumes that we can straightforwardly compare the entropy of two black hole solutions with thermodynamics defined at different coarse-graining scales k evaluated at the horizon. Performing

the integral in (65), and using the asymptotically safe RG running for G_k given by (30) together with (22), we find a logarithmic correction to the entropy, which we express as an entropy difference

$$S(A) - S(A_c) = \frac{A - A_c}{4G_N} + \frac{\pi}{g_*} \ln \left(\frac{A}{A_c} \right), \quad (66)$$

where A_c serves as a reference point. Clausius' entropy (66) is quite general in that it applies universally for rotating and charged black holes, despite of being only a function of the area A . The result falls back onto (1) in the limit of large areas where quantum corrections are subleading as $(\ln A)/A$. The main point is that (66) displays logarithmic quantum corrections linear in \hbar in addition to the terms found in (63). The sign of the proportionality factor is fixed by the sign of g_* for $A \geq A_c$. Note that for $g_* < 0$, a cancellation may occur between the two terms in (66) at $A \neq A_c$. Note also that the expression still requires input for $S(A_c)$ which is not determined by (65) alone. If we require that (64) holds true for both (63) and (65) at $A = A_c$, we find

$$S(A) = \frac{A}{4G_N} + \frac{\pi}{g_*} \left(1 + \ln \frac{A}{A_c} \right). \quad (67)$$

We conclude that the entropy (66), (67) derived phenomenologically using Clausius' rule differs by a logarithm from the expression (63) to which we were lead via thermodynamical considerations.

In order to gain further insights into an appropriate definition of the entropy, we also compute the statistical entropy of the RG improved metric obtained from the functional integral. This can be done using the “off shell” conical singularity method by S. Solodukhin [90] for the RG improved Schwarzschild black hole $J = q = 0$. To that end we approximate the Euclidean action by (8) plus the Gibbons-Hawking surface term, with $k = k_{\text{opt}}(A)$ according to (22). From this one obtains the free energy $F \equiv T \Gamma^E$, where Γ^E denotes the Euclidean effective action at the scale (22). Inserting the RG improved metric with (54) into the action we find that the free energy is given in terms of mass, temperature and entropy as

$$F = M - S T \quad (68)$$

for all RG scales. Here, the mass M is given explicitly by the mass function (23) (with $J = q = 0$), and the entropy is given by (24). From the validity of (68) we conclude that the statistical definition of entropy for the RG improved black hole space-time agrees with the thermodynamical definition of entropy, (24). This result also shows that the statistical definition of entropy differs by a logarithmic term from the one obtained by applying Clausius' rule.

D. Comparison

Finally, we compare our findings with selected results from the literature, see [91] for an overview. There is a vast body of work dealing with quantum corrections to black hole entropy, including applications of the conical singularity method in Euclidean space-times and relations to the conformal anomaly [92–96], the use of Cardy's formula [97], or studies of backreaction effects [98]. Logarithmic corrections to the entropy have also been found in the loop quantum gravity approach [99], perturbative quantum gravity and in string theory [100–104], or based on phenomenologically motivated expansions of classical space-times in powers of \hbar [105]. No general agreement has yet been achieved neither for the coefficient nor the sign of the logarithmic term, which has even been conjectured to be absent altogether [106].

In spite of this, some structural insights relate with our findings and are worth being highlighted. Firstly, for Clausius' entropy defined in (66) and (67), the logarithmic correction can in principle

have either sign depending on whether the RG running (30) is dominated by gravitational fluctuations or external matter fields. A positive (negative) sign correlates with the existence (absence) of a maximum black hole temperature. This very same link was noticed earlier in different settings, e.g. in [98] by analysing back-reaction effects, and in [105] by relating the logarithmic coefficient to the leading quantum correction for the surface gravity. Secondly, statistical considerations have been used in [107] to argue that the coefficient of the logarithmic term should be positive or vanishing. This constraint is in accord with our findings as long as gravity is anti-screening as implied by asymptotic safety. Finally, the sign pattern found here is in agreement with results from perturbative gravity [100] and the perturbative RG [101–103]. Within loop quantum gravity [108], the logarithmic correction has apparently the opposite sign, a result which has been interpreted as a hint towards an underlying non-perturbative fixed point [99].

The Wilsonian perspective developed here can also be applied for other UV completions of gravity. As such it may offer a common framework to study similarities and differences between approaches to quantum gravity.

VI. DISCUSSION

We have put forward an approach to understand the thermodynamics of black holes from a renormalisation group perspective, allowing for a continuous interpolation between black holes with largest and smallest mass. The main new ingredient is the scale-dependence of couplings such as Newton’s constant $G \rightarrow G(k)$ or the fine-structure constant $e^2 \rightarrow e^2(k)$ arising from the underlying effective action (8). We then find that thermodynamics imposes a relation between the RG scale and the horizon area of the black hole (22), which acts as a diffeomorphism-invariant cutoff for the effective action. The running couplings turn into functions of the black hole horizon area $G \rightarrow G(A)$ and $e^2 \rightarrow e^2(A)$ leading to modifications for the black hole equation of state. This is consistent with the view that the degrees of freedom with a characteristic wavelength set by the horizon scale, or smaller, are responsible for the thermodynamical nature of black holes. In this light, our set-up offers an interpretation for the emergence of black hole thermodynamics by treating the background gravitational field as a coarse-grained field which arises from modes with wavelengths bound by the horizon scale.

On a practical level, our set-up translates the RG-induced modifications into corrections for temperature, specific heat, and entropy without making assumptions about the actual RG running of couplings. If quantum gravity is (anti-)screening, we find that quantum corrections (decrease) increase the black hole temperature at fixed mass, charge, and angular momentum, as well as the entropy at fixed horizon area. It is conceivable that the broad picture, which relates the sign of the gravitational β -function with the sign of quantum corrections to temperature and entropy, persists in approximations beyond those adopted here. We also stress that our equations are completely general for stationary black hole solutions to Einstein-Maxwell gravity in four dimensions. For known classical black hole solutions, these are straightforwardly generalised to equations of state for black holes in dimensions different from four.

Provided, additionally, that gravity becomes asymptotically safe, the equation of state implies that the temperature is always smaller than the classical temperature for the same mass, angular momentum and charge. Furthermore, the temperature displays a maximum, and the specific heat of small black holes becomes positive, see Figs. 1 and 2. The new equation of state also predicts the existence of a lightest black hole for a causal horizon to exist. Interestingly, the weakening of gravity increases the entropy in comparison to the semi-classical result (1) for the same area, thereby enhancing the domain of validity for a thermal description (17) towards smaller black hole masses. On the other hand, the thermodynamical picture may be called into question for near-critical black

holes, where specific heat and temperature become small and the entropy parametrically of order unity. This regime would benefit from complementary studies.

We also showed that conformal scaling, a fingerprint of an RG fixed point, is encoded in the equation of state in the limit of vanishing horizon area. This regime can be viewed as the Cauchy horizon of the corresponding black hole space-time metrics. The result strengthens the view that asymptotic safety qualifies as a fundamental quantum theory for gravity. Furthermore, we have provided explicit space-time metrics which carry the same equation of state for all mass, charge or angular momentum. These findings close a gap in the study of RG-improved black hole metrics, showing that these can accommodate thermodynamical relations even in the charged and rotating case as long as the choice of RG scale is informed by the horizon area of the black hole.

Another interesting question relates to the quantum corrections for the entropy. For asymptotically safe gravity, we find that the entropy at fixed horizon area increases due to quantum corrections, leading to an entropy of the order of a few in the limit where the black hole becomes critical. The thermodynamical definition for entropy agrees with the statistical definition of entropy (68) for all RG scales and black hole masses, which serves as a consistency check. On the other hand, deriving the entropy using Clausius' rule leads to an additional term logarithmic in the area. Qualitatively, the difference arises from identifying the RG scale with the area before, or after, exploiting the effective equations of motion. It will be interesting to relate this difference to other definitions for the entropy, including entanglement entropy or Wald's entropy [84].

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